AN EVALUATION OF VICS'S BENEFIT BY USING MULTIPLE USER STOCHASTIC EQUILIBRIUM MODEL

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SUMMARY

This paper aims to develop a mathematical model that is able to forecast the traffic flow on the transportation network and evaluate the system benefits. This model is basically based on the ordinary multiple user network equilibrium model, but we can formulate the other equivalent mathematical optimization problem using an appropriate definition on VICS users share modeling. By using this model, we can measure the effectiveness that is consistent with the demand-modeling framework. Furthermore, we investigate the sensitivity not only of traffic flow but also of the share of VICS users by several numerical analyses. As a result, we can find the interest results that VICS information sometimes lessens the total travel time and that the social total cost can ameliorate even if we charge fee for VICS information.

INTRODUCTION

In Japan, development of ITS (Intelligent Transportation System) has already commenced as a means of drastically transforming traffic systems. Using advanced information and telecommunication networks as well as control technologies, ITS seeks organic integration and management of users, roads, vehicles and other transport means. ITS is expected to help resolve traffic congestion, accidents, and other road traffic problems, as well as improve the amenity and convenience for users. ITS consists of nine development areas. One of them is Vehicle Information and Communication Systems, so called VICS. This is thee most advanced area that promptly provides the latest necessary road traffic information to drivers via car navigation systems. The world's first VICS service started in Japan in April 1996. This system is composed of VICS-Unit, Beacons and VICS center. The most distinctive point compared with the ordinal navigation system is the two-way communication system. This system aims to improve the efficiency of its service so as to meet driver's requests, cut down travel costs by shortening the time of transportation, ensure safety by accurate information on the flow of road traffic, and preserve the environment by streamlining traffic. Furthermore, it aims to contribute to social and economic development. This system help drivers choose appropriate routes for smooth and comfortable driving, so that VICS is generally believed to be efficient in alleviating congestion and enhancing the performance of traffic networks.

However, lately VICS is about to become Dinosaur. The reasons are as follows;

(1) First, there is the difference in the type of beacons and contents of information that each agency adapts, so information is incompatible to each other. We cannot see the traffic conditions of expressway on main trunk roads.
(2) Second, the development of only devices and systems like VICS-Unit and Navigation system was conducted earlier than surveys on the relation between driver's behavior and traffic information. If this kind of surveys will not be conducted, we cannot decide when and where and what kind of traffic information will be provided.
(3) In addition, from the social and economic points of view, it seems that the provision of traffic information to drivers does not always have social benefit.
(4) So, we have to take some tools to evaluate the traffic information systems.

A fundamental requirement for VICS applications is the development of route choice behavior models with or without travel time information and the evaluation of the system and individual driver benefit. For the latter subject, we should study the travel demand forecasting under the provision of traffic information, and evaluation method of traffic information systems in the real world transportation networks. Recently there has been substantial development in modeling and evaluating VICS, however, it has not been sufficient. Then, we propose a normative mathematical model with which we forecast multiple route choice behavior equilibrium flow pattern with endogenous demand of VICS (Vehicle Information and Communication System) users. This model is basically based on the multiple user network equilibrium model provided by Yang (1998), but we can formulate the other equivalent optimization-programming problem using an alternative and appropriate definition on VICS users demand modeling. By using this model, we can measure the social benefit that is consistent with the demand-modeling framework. Furthermore, we obtain a very interesting paradox that the VICS information might not always produce social efficiency to the transportation. Furthermore, we find the useful results that total cost of the system is sometimes lower in the case when the VICS information cost is charged to drivers than in the case when the VICS information cost is free of charge.

NETWORK EQUILIBRIUM MODEL BASED ON MULTIPLE ROUTE CHOICE BEHAVIOR

Consider a network G=(N, A), where N is the set of nodes and A is the set of links in the road network. Let R and S be the set of origin and destination nodes. $r \in R$ and $s \in S$ are origin and destination node, respectively. We assume that the travel cost for each link, $a \in A$, is a function of the flow, x_a, on the link. This is described by an increase and strictly convex function of link flow, $t_a = t_a(x_a)$.

Suppose there are two types of drivers; one who uses and the other one who doesn't use the travel time information proposed by VICS on the common road network. When VICS can provide travel cost information to drivers, it is assumed that their perceptions of the travel cost improve and become more concentrated around the real travel costs. The drivers who use the VICS information will receive complete information on the minimum cost route, so they are able to find the minimum travel cost route in a deterministic user-optimal manner. Whereas the drivers who do not use the VICS information will only have incomplete information on route cost, and may choose alternatives which may not be in their best interest, and hence may take higher travel cost routes in a stochastic manner. Then, we can consider that drivers are assumed to have different perceptions of travel cost, so we describe the multiple traffic flow which results from these two route choice manners by mixed user equilibrium (UE) and stochastic user equilibrium (SUE) principle assignment. The difference between both behavioral principles is distinguished by the degree of the variance of travel cost perceptions of drivers. This multiple logit based route choice behavior model can be formulated as the following equivalent mathematical optimization program, which was proposed by Yang (1998):

$$Min: Z(\mathbf{x}, \mathbf{f}_1, \mathbf{f}_2) = \sum_{a\in A} \int_0^{x_a} t_a(\omega) + \sum_{g=1,2} \frac{1}{\theta_g} \sum_{rs} \sum_{k\in K} f_{g,k}^{rs} \ln(f_{g,k}^{rs} / q_{rs}^{g}) \tag{1}$$

$$s.t. \quad \sum_{k\in K} f_{g,k}^{rs} = q_{rs}^{g}, \quad \forall r\in R, s\in S, g = 1,2 \tag{2}$$

$$x_a = \sum_{rs} \sum_{k\in K_{rs}} \sum_{g=1,2} f_{g,k}^{rs} \delta_{a,k}^{rs}, \quad a\in A \tag{3}$$

$$f_{g,k}^{rs} \ge 0, \; \forall k\in K_{rs}^{g}, r\in R, s\in S, g = 1,2 \tag{4}$$

where g(=1,2) appears the driver's group whose level of perceptions is different. $f_{g,k}^{rs}$ is the route flow by driver's group g on route $k\in K_{rs}^{g}$. $\delta_{a,k}^{rs}$ is the link-route incident matrix which element is $\delta_{a,k}^{rs}$ =1 if route k between OD pair rs uses link a, 0 otherwise. q_{rs}^{g} is the demand of driver's group g between OD pair rs. This model results in two classes of SUE flow distribution with different degree of travel cost variability: one (g=1) is associated with the drivers who don't use the VICS information with a variability parameter θ_1 of the logit-based route choice model and the other one (g=2) is associated with the drivers who use the VICS information with a variability parameter θ_2. In this case, the constraint $\theta_1 < \theta_2 \to \infty$ should be kept between them.

It is clear that the optimization program (1)-(4) will lead to the multiple logit-based stochastic user equilibrium assignment models, because we can get the following logit-based route choice probability as the first-order optimality conditions of this program:

$$P_{rs,k}^{g} = \exp(-\theta_g c_k^{rs}) \Big/ \sum_{k\in K_{rs}} \exp(-\theta_g c_k^{rs}) \quad \forall k\in K_{rs}^{g}, r\in R, s\in S, g = 1,2 \tag{5}$$

where $c_k^{rs} = \sum t_a(x_a)\delta_{a,k}^{rs}$, being the actual travel cost on the k'th route. It is easy to conduct its proof, so we will neglect the details.

DEFINITION OF VICS USERS SHARE MODEL

We define the VICS users share model. We assume that the share of VICS users, shown by

g=2, is endogenous variable. As the same manner that Yang considers, this demand can be determined by the difference of the benefit, $S^1_{rs} - S^2_{rs}$, generated by the travel cost information system. It is natural that the share of VICS users assumes to be described by the following logit type model:

$$\Pr[2 \mid rs] = \frac{1}{1+\exp[\alpha + \beta(S^1_{rs} - S^2_{rs})]}, \quad r \in R, s \in S \tag{6}$$

where α, β are parameters that should be estimated. We will show two methods to define the benefit derived from VICS.

Definition By The Travel Cost Saving

As the benefit, Yang used the average travel cost shown in the following equation:

$$S^g_{rs} = \sum_{k \in K_{rs}} P^g_{rs,k}(c_{rs}(\mathbf{f}_1, \mathbf{f}_2)) \cdot c^{rs}_k \quad (g = 1,2) \tag{7}$$

In Eq.(7), we need to calculate the route cost. However, it is difficult to eliminate all routes in the large-scale road network. We will propose an effective method which is able to calculate the expected value of travel cost by using link flow by rs and g, $x^g_{a,rs}$, obtained from the Dial's backward procedure as follows:

$$S^g_{rs} = \sum_{a \in A} \sum_{rs} x^g_{a,rs} \cdot t_a(\sum_{rs} \sum_{g=1,2} x^g_{a,rs}) / q^g_{rs} \quad (g = 1,2) \tag{8}$$

Definition By The Improvement Of Uncertainty

As the index of benefit, we will introduce the following uncertainty function which means the uncertainty level of travel cost between OD. For logit-based route choice model as shown in Eq. (5), the uncertainty function is

$$S^g_{rs} = \min_{k \in K_{rs}} \{c^{rs}_k\} - \left\{ -\frac{1}{\theta_g} \ln \sum_{k \in K_{rs}} \exp(-\theta_g c^{rs}_k) \right\} \quad (g = 1,2) \tag{9}$$

This definition is consistent with the VICS share and route choice demand models which are based on the random utility theory, so it should be much useful for the evaluation of VICS.

EQUIVALENT MATHEMATICAL OPTIMIZATION MODEL

When we use the demand function with the assumption of benefit (8), the equilibrium level of VICS user share becomes an endogenous variable to be determined as same as multiple behavior equilibrium flow. However, we have been unable to introduce an appropriate stochastic user equilibrium model that is formulated by mathematical program for the above multiple behavior equilibrium model with elastic VICS user share demand. Yang proposed a general parametric nonlinear programming problem and investigated its existence, uniqueness and stability. Furthermore, he proposed a revised solution algorism, so called the mixed equilibrium assignment algorithm (MEAA), based on the standard method of successive average (MSA) where Dial's algorithm is used to find a decent direction of the objective function without path enumeration. When we calculate the benefit by Eq. (8), this algorithm is much useful because we can obtain link flow by *rs* and g, $x_{a,rs}^{g}$ by using Dial's algorithm.

In the case that we use Eq. (9) as the VICS share demand function, we can find the equivalent mathematical optimization program as follows:

$$Min{:}\ Z(\mathbf{x},\mathbf{f}_1,\mathbf{f}_2,\mathbf{q}^1,\mathbf{q}^2)$$

$$= \sum_{a\in A}\int_0^{x_a} t_a(\omega)d\omega + \sum_{g=1,2}\frac{1}{\theta_g}\sum_{rs}\sum_{k\in K_{rs}} f_{g,k}^{rs}\ln(f_{g,k}^{rs}/q_{rs}^{g})$$

$$-\frac{1}{\beta}\sum_{rs}\int_0^{q_{rs}^2}(\ln\frac{\omega}{\bar{q}_{rs}-\omega}+\alpha)d\omega \tag{10}$$

$$s.t.\quad \sum_{g=1,2} q_{rs}^{g} = \bar{q}_{rs},\quad \forall r\in R, s\in S \tag{11}$$

$$\sum_{k\in K_{rs}} f_{g,k}^{rs} = q_{rs}^{g},\quad \forall r\in R,\ s\in S,\ g=1,2 \tag{12}$$

$$x_a = \sum_{rs}\sum_{k\in K_{rs}}\sum_{g=1,2} f_{g,k}^{rs}\delta_{a,k}^{rs},\quad a\in A \tag{13}$$

$$q_{rs}^{g}\ge 0,\quad \forall r\in R,\ s\in S,\ g=1,2 \tag{14}$$

$$f_{g,k}^{rs}\ge 0,\quad \forall k\in K_{rs},\ r\in R,\ s\in S,\ g=1,2 \tag{15}$$

where $\bar{q}_{rs}$ is the total demand between OD pair *rs* and is given. In this model, not only path flow but also the demand of VICS users, q_{rs}^{g}, becomes unknown variable. This is one of the stochastic user equilibrium models with elastic modal choice demand. It is clear that the solution of this model is satisfied with Eqs. (5), (6) and (9) because the first order conditions of this model are satisfied with these all equations, so we skip the detailed proof.

SOLUTION ALGORITHM

We can apply the ordinal solution algorithm performing MEAA (the Mixed equilibrium assignment Algorithm) within the updating process of VICS user demand. We show the procedure to solve the above optimization model:

Step 0: Set an initial value for the VICS user demand, $q_{rs}^{2(n)}$, and thus

$$q_{rs}^{1(n)} = \bar{q}_{rs} - q_{rs}^{2(n)},\quad r\in R, s\in S.\qquad \text{Set } n{:=}0.$$

Step 1: Conduct MEAA.

Step 1-0: Perform an all-or-nothing loading for $q_{rs}^{2(n)}$ and a stochastic network loading for $q_{rs}^{1(n)}$ based on a set of initial free-flow travel time $t_a^{(k)}$. Find sets of link flows of both groups, and $x_a^{(k)} = \sum_{g=1,2} x_{g,a}^{(k)}$. Set k:=0

Step 1-1: Update link cost $t_a^{(k)} = t_a\left(x_a^{(k)}\right)$. For the updated link cost, perform the Dial's Forward Pass process and find a set of link weight of both groups, $\{W_{g,a}\}$.

Step 1-2: Find the set of auxiliary link flow $y_a^{(k)} = \left\{y_{g,a}^{(k)}\right\}$ by performing an all-or-nothing loading for $q_{rs}^{2(n)}$ and a stochastic network loading for $q_{rs}^{1(n)}$ based on a current set of link travel cost $t_a^{(k)}$.

Step 1-3: Update the new flow pattern of both groups by setting

$$x_{g,a}^{(k+1)} = x_{g,a}^{(k)} + \frac{1}{k}\left(y_{g,a}^{(k)} - x_{g,a}^{(k)}\right)$$

Step 1-4: If some convergence conditions on $x_{g,a}^{(k+1)}$ are satisfied, then go to *Step 2*. Otherwise, let k:=k+1 , and go to *Step 1-1*.

Step 2:Calculate $S_{rs}^{g(n)}$, $r \in R, s \in S$, g=1,2 by Eqs.(8), (9), and the set of auxiliary VICS users demand by

$$q_{rs}^{2(n)} = \bar{q}_{rs} \cdot \frac{1}{1+\exp[\alpha + \beta(S_{rs}^{1(n)} - S_{rs}^{2(n)})]}, \quad q_{rs}^{1(n)} = \bar{q}_{rs} - q_{rs}^{2(n)}, \quad r \in R, s \in S$$

Step 3: If $\left|q_{rs}^{2(n+1)} - q_{rs}^{2(n)}\right| \le \varepsilon$ for all $r \in R, s \in S$, where ε is a predetermined tolerance value, then stop. Otherwise let n:=$n+1$ and go to *Step 1*.

NUMERICAL ANALYSIS

Here are some numerical examples that will be presented to investigate how the degree of the perceived travel cost variability of drivers, and congestion level on the network and so on affect the economic benefit of VICS and share of VICS users. The model network, that is the same as Yang's one, is shown in **Figure 1**. As the link performance function, we use the BPR type function which parameters are shown in Figure 1 as well. It is assumed that there is only one OD pair from node-1 to node-12 in the base case. The sensitivity parameters of VICS

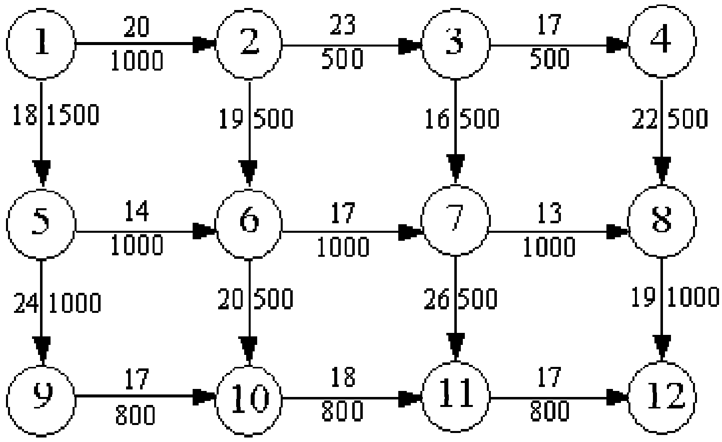

Figure 1. Model Network and Link Performance

users share function are assumed to be $\alpha = 1.75$, $\beta = 0.50$. In only the case that we define the user's benefit by improvement of driver's uncertainty, we should use. $\beta = -0.50$. Those values are the same as Yang's ones.

We will show some basic attributes of this model by following some numerical analyses. First is the sensitivity of VICS users share against the change of the total number of trips and the short distance trip ratio in total trips. The short distance trip is the trip between node-1 and node-7. A smaller value of θ_1 implies a higher degree of drivers' perceived travel cost variability, and thus the VICS information might produce more utility to the VICS users. Of course, we assume that the value of θ_2 is infinite. However, when we use the average travel time as the definition of a VICS benefit, the VICS users share decreases as the total number of trips increases. On the other hand, when we use the uncertainty function value as a VICS benefit, there is the opposite tendency. The second is the sensitivity of the VICS users share against the short distance trips ratio in the total number of trips. As the short distance trips from node-1 to node-7 increases, VICS users ratio increase in the case that we use the average travel cost as the VICS benefit. In the case that the uncertainty function is used as the definition of VICS benefit, there is the opposite tendency. The VICS might produce more benefit to the longer distance trips, so the uncertainty value seems to be appropriate as the index that we defined in the VICS benefit. From these results of analyses, it is good to say

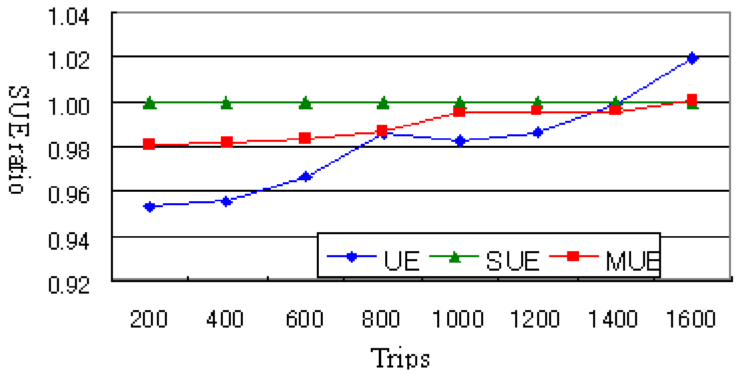

Figure 2. Total Travel Time

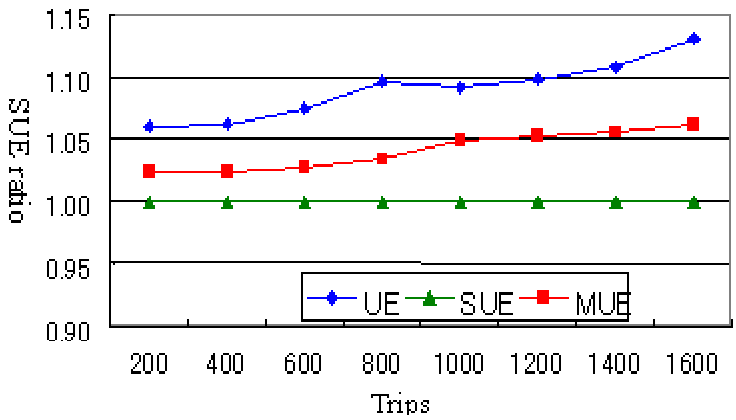

Figure 3. Satisfaction Function Value

that the second definition regarding the user's share model has better attributes than the first one.

Figures 2 and **Figure 3** show the change of two kinds of indices that evaluate the efficiency of this whole transportation system. First index is the total travel time and the other is the sum of uncertainty value. These indices are shown against the total number of trips under three different equilibrium assignment methods such that all trips are assigned by deterministic user equilibrium (UE) and stochastic user equilibrium (SUE) and multiple user stochastic equilibrium (MUSE) model. These values of UE and MUSE are standardized by the value of SUE. These values increase monotonously as the total number of trips increases. For the sum of uncertainty value shown in Figure 5, the reasonable result such that UE>MUSE>SUE is kept. On the other hand, the total travel time by UE and MUSE is bigger than by SUE when the total number of trips is many in **Figure 4**. This result means a paradox, that the VICS information might not always produce social efficiency to the transportation system in case of heavy traffic congestion when we evaluate the social efficiency by used total travel time.

When we assume the cost that the drivers need to get the VICS information, the uncertainty function of VICS uses is written as follows;

$$S_{rs}^{2} = -\frac{1}{\theta_2} \ln \sum_{k \in K_{rs}} \exp(-\theta_2 c_k^{rs}) - C_{info} \tag{16}$$

where C_{info} is the VICS information getting cost. This unit is changed into the unit of uncertainty value. Figure 5 shows the change of the total cost of this transportation system against the cost of the VICS information under the total number of trips. The values are shown by the ratios to the value in the case of free VICS information cost. From this figure, we find that there is a maximum point of total cost, which is the worst cost of the VICS information from a viewpoint of total travel cost for the transportation system. Furthermore, when the total number of trips is 1,600, the total cost of the system is lower than when the VICS information cost is free of charge.

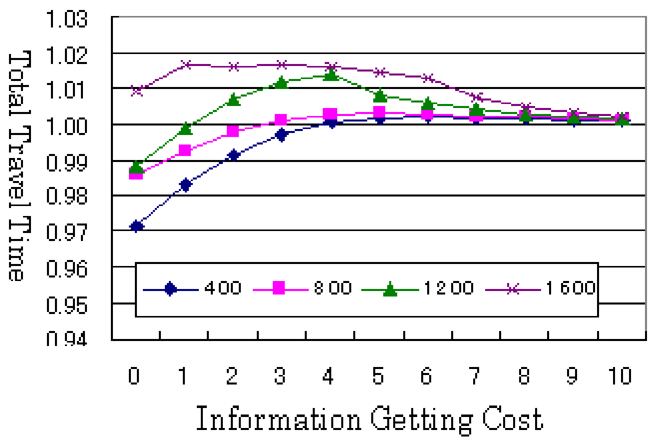

Figure 4 Total Travel Time with Information

CONCLUSIONS AND FURTHER EXTENSIONS

We conclude our studies and show some further research subjects;

(1) We have proposed a multiple users stochastic equilibrium model with elastic VICS users demand. The VICS users demand is defined by the travel time saving and travel cost accuracy improving of driver. The latter model is formulated by a mathematical programming model based on the drivers' route choice and VICS information usage choice behavior.

(2) Through some numerical analyses, there are some different characteristics between them in the change of equilibrium VICS users share against the traffic congestion level.

(3) We obtain a very interesting paradox that the VICS information might not always produce social efficiency to the transportation.

(4) Furthermore, we find the useful results that total cost of the system is sometimes lower in the case when the VICS information cost is charged to drivers than in the case when the VICS information cost is free of charge.

Now, we attempt to expand our model and numerical sensitivity analysis into the inter-modal transportation system, because the advanced navigation systems like a VICS intend to provide the route guidance traffic information not only for cars but also for public transportation in order to use the inter-modality.

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